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A Combinatorial Auction with Multiple Winners for COLR

Frank Kelly and Richard Steinberg
University of Cambridge

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Executive Summary

We describe a discrete-time auction procedure called PAUSE (Progressive Adaptive User Selection Environment) for use in assigning COLR (Carrier of Last Resort) responsibility. The PAUSE auction is based on and generalises the auction structure used for PCS (Personal Communications Services) licensing; in particular, the PAUSE auction proceeds by discrete rounds within stages driven by progressive activity rules. In addition, the auction incorporates synergies by allowing for every possible combinatorial bid; in particular, it incorporates the use of AUSM (Adaptive User Selection Mechanism) bidding, which has been tested in the laboratory at the California Institute of Technology.[1] By proceeding by discrete rounds in stages with progressive activity rules, the threshold problem and deception ('snake-in-the-grass') effect are lessened, and at the same time the computational effort required of the bidders is reduced. The auction also allows for multiple winners.

The inherent computational complexity of combinatorial bidding cannot be eliminated; however, the auction procedure described here is computationally simple for the auctioneer and thus is very efficient to run. The computational burden of evaluating synergies rests with the bidders claiming those synergies, while the auctioneer simply checks that a bid is valid. There is very little computational burden for small players interested in only a small number of assets. If *no* synergies are claimed, then the auction reduces to the auction of the type utilised for the PCS licenses.

[1] Bykowsky, M.M., R.J. Cull, and J.O. Ledyard, 'Mutually Destructive Bidding: The FCC Auction Design Problem', *Social Science Working Paper* 916, California Institute of Technology, 1995.

1. Overview of the Auction

Define a **PAUSE (Progressive Adaptive User Selection Environment) Auction** to be a two-stage auction with:

(i) In Stage 1, 'PCS-type' activity rules, i.e., three substages with progressive eligibility requirements and an improvement margin (bid increment) requirement;

(ii) In Stage 2, AUSM bidding in two substages with progressive eligibility requirements and and an *exact* improvement margin requirement;

(iii) No bid withdrawals and no bid waivers.

The PAUSE auction is designed to be fully general in that every possible combinatorial bid is available to the bidders. If, however, the auctioneer wishes to restrict the bids in any manner that he finds convenient to verify, the auction structure will accommodate this, and the auctioneer can announce to the bidders a list of attributes a bid must have. (An example of such an attribute might be: 'bids that are combinatorial are to be composed of geographically contiguous subsets of the properties'.) This is formalised in the next section.

2. Definitions

Label *properties* $j \in J$, and *blocks* $k \in K$, where $K = K(J, A)$ is a set of subsets of J defined by a set of *attributes* A that are computationally simple for the auctioneer to verify for each member of K. Let

$$K_n = \{k \in K(J,A): 1 \leq k \leq n\},$$

where $|k|$ is the number of properties in block k.

(Thus, K_1 is the set of blocks allowed by the attribute set and consisting of a single property, K_2 is the set of allowed blocks consisting of at most two properties, K_3 is the set of allowed blocks consisting of at most three properties, and so forth.)

A *partition* $P = (p_1, p_2, \ldots, p_r)$ is a collection $p_1, p_2, \ldots, p_r \in K$ such that $\bigcup_{i=1}^{r} p_i = J$, and $p_i \cap p_j = \varnothing$, $i \neq j$.

(In words, a partition is a grouping of all the properties in the auction into sets that do not overlap.)

A *composite bid* comprises a partition $P = (p_1, p_2, \ldots, p_r)$ together with an *evaluation*

$$(C(P); c(p_1), c(p_2) \ldots, c(p_r))$$

where

$$C(P) = \sum_{i=1}^{r} c(p_i), \qquad (*)$$

and $c(p_i)$ is the *bid* for block p_i.

To be more precise, $c(p_i)$ is *the value of the bid for block* p_i. A composite bid consists of $3r+1$ pieces of information, capable of registration in a database. The first piece of information is the total value of the composite bid, $C(P)$. The $3r$ pieces of information are, for each i ($i = 1, 2, \ldots, r$): (1) the specification of the block p_i, (2) the value of the bid on the block, $c(p_i)$, and (3) the identity of the bidder for block p_i.

Note that $c(p_i)$ is the *total subsidy for block* p_i. It corresponds to a *subsidy per subscriber in block* p_i of $c(p_i)/\|p_i\|$, where $\|p_i\|$ is the total number of subscribers in all the properties in p_i.

Items (1) and (2) are available from the database to all bidders; item (3) may be available only to the auctioneer and the bidder concerned, or may be public information.

3. The Procedure

Opening Bids

This analysis does not attempt to determine the merits of historical versus forward-looking cost models. However, opening bids for each property could be the lower of the historical cost and the forward-looking cost for that property. (By 'forward-looking cost' we mean, for example, the Total Service Long Run Incremental Cost or the cost obtained from the Benchmark Cost Model.)

If the lower of these two costs is the historical cost, then it is announced that historically service has been provided on this property at a certain subsidy level and it is expected that service will be provided at no higher than that level in the future.

If, on the other hand, the lower of these two costs is the forward-looking cost, then forward-looking cost would serve as a starting point for our analysis to determine the minimum subsidy to provide service in a given market.

Stage 1 - Bidding on Individual Properties.

The Bidders: Each bidder submits a collection of bids on individual properties.

The Auctioneer: In each round, for each property the auctioneer checks that a bid on that property is *valid* by verifying that it decreases the value of the last accepted bid on that property by *at least* the specified bid increment. In each round, the lowest valid bid on each property is accepted. The round ends when bidding ends on all properties. Stage 1 is divided into three substages that correspond to the stages of the PCS auctions. At the conclusion of the third substage, the leading (i.e., lowest) bids on the properties are registered to their respective owners, and the auctioneer announces the number of multiple winners that will be accepted and necessary for property j, as determined by the rule described below.

Activity Rules: A bidder is *active* on a property if he has the leading bid from the previous round or submits an acceptable bid in the current round. Each of the three substages contains an unspecified number of bidding rounds. The bidders must remain active on properties covering, respectively in the three stages, 60 per cent, 70 per cent, and 80 per cent of the number of subscribers for which they wish to remain eligible to bid. (In this document, by *subscribers* we mean 'subscribers counted under the universal service provisions for support for high cost areas'.) The transition from substage 1 to 2 occurs when there are bids on no more than 10 per cent of the subscribers for three consecutive rounds, from substage 2 to substage 3 when there are bids on no more than 5 per cent of the subscribers for three consecutive rounds.

Bid Increments: In each round there is an *improvement margin requirement*, viz., a new bid must improve on the previous bid on that property by a specified amount. The specification of this amount can follow the rules used in the PCS auctions.

Multiple Winners: At the conclusion of the third substage, after the leading bids on the properties are registered to their respective owners, the auctioneer announces the number of winners on each property j as follows:[2] (1) if at least one competing bid is within 15% of the lowest bid, then all who bid within 15% of the lowest bid are designated as winners; (2) if no competing bid is within 15% of the lowest bid but one is within 25%, then the two lowest bidders are winners, and (3) if no bid is within 25% of the lowest bid, then there is a single winner, viz., the lowest bidder. The number of multiple winners on each property j at the end of Stage 1 is denoted by $k(j)$.

Before the start of Stage 2, property j is replaced by $k(j)$ properties $j_1, j_2, j_3, \ldots, j_{k(j)}$.

[2] This is the formula proposed by Paul Milgrom in his attachment to GTE's Comments. Other formulas for multiple winners are possible.

Stage 2 - Combinatorial Bidding

The Bidders: Each bidder submits a single composite bid on a collection of properties, where each bidder's partition $P = (p_1, p_2, ..., p_r)$ is restricted to $p_i \in K_n$, where $c(p_i)$ is either a new bid for block i, or a registered bid. Initially, $n = 2$. For a composite bid to be valid, for each property j the bid must not allocate j_s and j_t $(s \neq t)$ to the same player. In this stage of the auction, the bidder identities are to made public. Thus, the validity of a composite bid--and in particular the requirement that the bid does not allocate j_s and j_t $(s \neq t)$ to the same player--can be checked by the player constructing the composite bid.

The Auctioneer: In each round, the auctioneer checks that a composite bid is *valid* by checking:

(i) Bid Validity: each bid claiming to be registered is indeed registered in the database; that new bids satisfy $p_i \in K_n$, that is, that new bids are on allowed blocks of not more than n properties; and, for each property j, the composite bid does not allocate j_s and j_t $(s \neq t)$ to the same player

(ii) Evaluation Validity: equation (*) holds, i.e., the value $C(P)$ of the composite bid is indeed the sum of the bids on each of its blocks, and

(iii) Increment Validity: bid $C(P)$ is less than the last accepted bid by *exactly* the specified bid increment.

In each round of Stage 2, the new collection of bids on the blocks $\{c(p_i)\}$ are registered to their respective owners, and the lowest valid composite bid is accepted. The round ends when bidding ends. Stage 2 is divided into two substages.

Activity Rules: A bidder is *active* on a property if his bid on a block containing that property forms part of the accepted composite bid of the previous round, or if he submits a valid bid in the current round on a block containing that property. Each of the two substages contains an unspecified number of bidding rounds. The bidders must remain active on properties covering, respectively in the two stages, 90 per cent and 95 per cent of the number of subscribers for which they wish to remain eligible to bid. The transition from substage 1 to substage 2 occurs when there are bids on no more than 10 per cent of the subscribers for three consecutive rounds.

Bid Increments: In each round there is an *exact improvement margin requirement*:

If $c(p_1), c(p_2)...., c(p_s)$ are the *new* bids in a composite bid, then the evaluation must improve on the previous best evaluation by *exactly* εs, i.e., an improvement of ε per block on average.

Multiple Winners: At the conclusion of Stage 2, the $k(j)$ winners on property j are each designated a $1/k(j)$ share of the responsibility on property j. Specifically, the contractual obligation carried by each player is as follows: *The player will receive his bid subsidy per subscriber on* up to $1/k(j)$ *of the total number of subscribers in that property, and he is required to serve* at least $1/k(j)$ *of the subscribers in that property.* The particular subscribers that make up this fraction are not specified; the player will *compete* for these subscribers with the other winners on that property. If a subscriber is unserved in a property with multiple winners, the regulatory authority may require any one of the multiple winners who is not serving the full amount of his contractual share to serve that subscriber. (There is thus a considerable incentive for players to actively seek to serve their share of subscribers, lest they be required to serve subscribers not of their choice.)

4. Other Auction Rules

Bid Withdrawals

No bid withdrawals are allowed in either stage.

In the PCS auctions, bid withdrawals were permitted. Specifically, a high bidder withdrawing his bid during the course of the auction was required to pay the difference between his bid and the price for which the licence ultimately sold; a winning bidder withdrawing after the close of the auction suffered an extra penalty. It may be asked why bid withdrawals were permitted, since they complicate the auction. Paul Milgrom, in his attachment to GTE's Comments[3], clearly states the motivation: 'In effect, a bid withdrawal substitutes partially and quite imperfectly for combinatorial bidding.'

Bid Waivers

For simplicity, there are no bid waivers in either stage.

[3]Statement of Paul R. Milgrom Attached to GTE's Comments in Response to Questions, CC Docket No. 96-45.

5. Discussion

Bid Increment and Block Size

McAfee and McMillan (1996)[4] report that in the MTA auction[5] (in which the highest bid won the licence), aggressive bidding in early rounds took the form of 'jump bidding': entering bids far above the required minimum bid increment. Analogously, jump bidding in the COLR market would mean entering bids far below the required minimum bid increment. In a combinatorial auction, jump bidding for a block of several properties would be effective at preventing small players from piecing together a comparable composite bid (the threshold effect). The rule that the improvement margin must be an exact increment is designed to lessen the threshold effect. It also helps keep the computation requirement down, by limiting the ranges of possibilities that need to be considered by bidders.

The size of the bid increment ε and the rate of increase of the block size limit, n, are used by the auctioneer to control the speed of the auction, in conjunction with the activity rules. For example, the auctioneer might move n from the starting value of 2, to 3, 4, 5, ...; however the auctioneer might instead move n to 4, 8, 16,... , In either case the value of the bid increment would decrease, and the activity rule percentage increase, as n increases.

Multiple Winners

Several variants are possible on how multiple winners are determined. For example, suppose that the auctioneer, before the start of Stage 2, decides that $k(j)$ is an upper bound on the number of multiple winners for property j, and that $h(j)$ is a lower bound on the number of multiple winners for property j. Then composite bids may allocate both and to the same player, giving that player a $2/k(j)$ share of property j, provided the auctioneer's announced restriction is satisfied by the composite bid. Of course, composite bids may allocate more than two replicated properties to the same player, resulting in a share of that property to the player that is even larger.

For example, if $k(j) = 5$ and $h(j) = 3$, then there must be at least 3 winners on property j, and a winner could have a 1/5, 2/5 or 3/5 share of property j. More generally, if property j has an upper bound of $k(j)$ and a lower bound of $h(j)$ on the number of multiple winners, than a winner could have between $1/ k(j)$ and $(k(j)-h(j)+1)/ k(j)$ share of property j.

It is essential that, before the start of Stage 2, the auctioneer specifies the rules that need to be satisfied by a valid composite bid in a manner that can be checked by players, as well as by the auctioneer. In particular, the auctioneer should not attempt to decide the number of multiple winners after Stage 2, since to do so would involve the auctioneer in a task of some considerable computational complexity.

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[4] McAfee, R.P. and J. McMillan, 'Analyzing the Airwaves Auction', *Journal of Economic Perspectives* 10, 1, 1996, 159-175.

[5] The MTA auction ran from December 1994 to March 1995 and sold broadband licenses covering the 51 'Major Trading Areas', or MTAs, into which the United States is divided.

Contractual Obligation and Price

If a fixed number of multiple winners will be accepted on a given property, and the contracts for each will carry the same contractual obligation, then rational behaviour by the bidders will generally lead to them achieving the same price (within ε) on successful bids on blocks comprising just that property. This is simply the law of one price, i.e., a bidder is unlikely to pay more for something identical available at a lower price. Of course the bounded rationality of players, together with the inherent computational complexity of combinatorial bidding, may cause bidders to occasionally depart from the law of one price. Note also that a price for a property cannot be determined from a composite bid if within that composite bid the property is part of a larger block. Similarly, if the contracts carry different obligations, then rational behaviour by the bidders will lead to a variety of achieved prices reflecting the bidders' views about the value to the bidders of the various obligations.

Computational Complexity

The computational complexity of Stage 2 is discussed in the Technical Appendix.

Technical Appendix: Computational Complexity of Stage 2

Number of Rounds

Since in each round the value of the accepted composite bid must decrease by at least ε over the previously accepted composite bid, the number of rounds in total is bounded above by $C_0(P_0)/\varepsilon$, where $C_0(P_0)$ is the value of the opening composite bid (perhaps set by the auctioneer).

Number of Registered Bids

Let B be the number of bidders. Since each bidder is allowed to make at most one composite bid per round, the maximum number of bids that needs to be registered by the auctioneer is bounded above by

$$\frac{C_0(P_0)}{\varepsilon} B|J|.$$

Discussion

In general, it may be an NP-complete problem for a bidder to determine whether he can make a composite bid that beats the currently accepted composite bid. The results of Rothkopf *et al.* (1996)[6] show that, if the form of composite bids is restricted in one or other of several possible ways, then the problem becomes manageable. However bidders are unlikely to agree upon the form of the appropriate restriction on composite bids. We view the elicitation of the form and size of potential synergies as a major purpose of the auction.

Work on computationally difficult problems shows that in several situations where finding the exact optimum is hard, finding a good approximation to the optimum with high probability may be relatively easy (Jerrum and Sinclair 1996)[7]. It is our belief that the traditional problems of elicitation and gaming are more serious difficulties than the possible computational burden on those bidders claiming complex synergies.

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[6] Rothkopf, M.H., A. Pekec, and R.M. Harstad, 'Computationally Manageable Combinational Auctions', RUTCOR, Rutgers University, May 1996.

[7] Jerrum, M. and A. Sinclair, 'The Markov Chain Monte Carlo Method: An Approach to approximate Counting and Integration', in Dorit S. Hochbaum (ed.), *Approximation Algorithms for NP-Hard Problems*, PWS Publishing Company, Boston, Massachusetts, 1996.